

Testing cosmological defect formation in the laboratory*

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Abstract

Topological defects such as cosmic strings may have been formed at early-universe phase transitions. Direct tests of this idea are impossible, but the mechanism can be elucidated by studying analogous processes in low-temperature condensed-matter systems. Experiments on vortex formation in superfluid helium and in superconductors have so far yielded somewhat confusing results. I shall discuss their possible interpretation.

1 Introduction

For quite a few years, my research has been mainly on the interface between particle physics and cosmology. So it may be quite surprising that I am talking at a conference on Vortex Matter! But there are good reasons. New connections have been forged in the last few years with condensed matter physics — connections in which vortices play a central role, and that form the theme of the ESF Programme on *Cosmology in the Laboratory*. I want to explain how this came about. (I regret that Grisha Volovik, who co-chairs that Programme with me, and who hoped to be here too, was unable to come.)

Our present understanding of fundamental particle physics leads us to believe that very early in its history, the Universe underwent a series of phase transitions. The full symmetry of the underlying theory is apparent only at extremely high energies. As the Universe expands and cools, the symmetry is progressively broken. For example the *electroweak* symmetry $SU(2) \times U(1)$ is manifest only at energies above a few hundred GeV; below that the symmetry is broken by the Higgs mechanism to the $U(1)$ of electromagnetic gauge

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invariance. At even higher energies, the symmetry may be larger still. There may be a *GUT* transition at an energy scale of about 10^{15} GeV, above which the strong, weak and electromagnetic interactions are all combined in a *grand unified theory* with a symmetry group such as $SU(5)$ or $SO(10)$. If, as is widely believed, the underlying theory is a *superstring* or *M-theory*, the implied supersymmetry must be broken at a transition scale of perhaps a few TeV.

But how can we test these ideas? The energy scales are far too high to be accessible to accelerator experiments. The only place such energies are found is the early Universe, in the first fraction of a second after the Big Bang. But of course the early Universe was opaque; we have no direct observational access to it. We have to look for surviving traces of these very early events. There are various possibilities, but one in particular that I want to talk about. A common feature of symmetry-breaking phase transitions is the formation of topological defects of one kind or another: monopoles, cosmic strings or domain walls. All these have analogues in condensed-matter systems: hedgehogs, vortices or flux tubes, and solitons.

Because of their topological stability, defects may have survived long enough to be observable [1–3]. Indeed, monopoles and domain walls, if formed, could have survived all too well, and already be in conflict with observation, though there are possible ways out of this. Cosmic strings, however, are attractive from a cosmological perspective. They might help, for example, in explaining *baryogenesis* — creating the observed matter–antimatter asymmetry of our Universe — or *magnetogenesis* — seeding the magnetic field of galaxies.

To make definite predictions that can be tested against astronomical observation, we need to answer several questions:

- What *kinds* of defects can be formed in the hypothesized phase transitions?
- How *many* defects would be formed at the phase transition?
- How would they *evolve* as the Universe expands?
- How would they *interact* with matter and radiation to generate observable signatures?

We have what we believe are good answers to these questions, based on various assumptions and approximations. But can we rely on them, when our methods are untested?

Cosmology is an exciting subject, but it suffers from one major shortcoming: we have no means of conducting controlled experiments, so we cannot directly test our calculational methods. It is here that the analogy with condensed matter comes in. The mathematical descriptions of topological defects in particle-physics models and in condensed-matter systems are often very similar. The last of the four questions above is a purely cosmological one, but for the others analogies with condensed matter may be very instructive.

Our methods of computing, for example, the number of defects formed at a cosmological phase transition can also be applied to a transition in a suitably chosen condensed-matter system, on which we can do real experiments, an idea first suggested by Zurek [4]. Following through this idea has led to some very exciting developments in condensed-matter physics.

2 Cosmic strings

The simplest model in which strings or vortices are formed is described by a complex scalar field ϕ with U(1) symmetry, $\phi \rightarrow \phi e^{i\alpha}$. This may be a global symmetry or a local gauge symmetry. In the latter case, ϕ interacts with a gauge potential A_μ transforming according to $A_\mu \rightarrow A_\mu - \frac{1}{e}\partial_\mu\alpha$. The scalar potential V must be U(1)-invariant. If we choose

$$V(\phi) = \lambda(|\phi|^2 - \eta^2)^2,$$

(often called the *Mexican hat* potential), this is the Abelian Higgs model, the relativistic analogue of the Ginzburg–Landau model. In this case, $\phi = 0$ is a maximum of the potential, so the symmetry is broken. There is a degenerate ground state, labelled by the phase angle α : $\langle\phi\rangle = \eta e^{i\alpha}$.

This model (with or without the gauge field) exhibits a phase transition. Above a critical temperature $T_c \sim \eta$ (I use units in which $c = \hbar = k_B = 1$) there are large fluctuations in ϕ about a mean value of zero. As the system is cooled through the transition, ϕ falls into the trough of the potential and acquires a nonzero average value. In so doing it has to choose a phase α . But in a large system such as the Universe, there is no reason why this choice should be the same everywhere; α will vary randomly in space. Indeed, there can clearly be no correlation between the directions of α in regions beyond the causal horizon, which have had no previous causal contact.

Now it may happen that if we traverse some large loop in space, the value of α will change by 2π (or some multiple of 2π). In that case, somewhere inside the loop ϕ must go through zero. Indeed, it must vanish along a curve that threads its way through the loop; this is the cosmic string. It is the precise analogue of the Abrikosov vortex in the Ginzburg–Landau model.

Because ϕ has to climb over the central hump of the potential, there is excess energy trapped on the string. In the local-symmetry case, this yields a *string tension* of order η^2 . When the symmetry is global, the energy per unit length of an isolated string is logarithmically divergent, but in a real system the divergence is cut off at the average string separation (or the system size). The strings are *topologically stable*. Strings will tend to shorten with time, under

the effect of the string tension, and small closed loops of string may shrink and disappear, but they cannot break (except by a very exotic and unlikely process involving the formation of a pair of black holes). Because α is uncorrelated over large distances, the phase transition will generate a random tangle of cosmic string. The density of string will tend to decrease with time, but some string is likely to survive a long time, long enough to have observable consequences.

This $U(1)$ model is merely a simple example, but strings also appear in many more realistic models, in particular in grand unified theories. Their dynamics and likely evolution are not usually very different, however — though there is one caveat: some strings can support persistent currents carried by fermions trapped in zero modes on the string, strongly influencing their behaviour.

3 The Zurek predictions

As the Universe cools through the relevant transition, we expect a random tangle of string to be formed, with some characteristic scale ξ_{str} . In other words, in any randomly chosen volume ξ_{str}^3 , we expect to find on average a length ξ_{str} of string. The question is: what is it that determines this scale? Of course, it must be related in some way to the correlation length ξ of the scalar field. But in the neighbourhood of the transition, ξ is changing very rapidly. Indeed, at a second-order transition, the equilibrium correlation length ξ_{eq} goes to infinity at the transition temperature T_c . So this raises the question: at what temperature, or what time, should we equate ξ_{str} to ξ ?

Zurek [5,6] has provided an answer to this question based on a causality argument (see also [7]). When the system goes through a real transition, at a finite rate, it is clear that ξ cannot become infinite. There is a maximum speed, c , with which correlations in the phase of the scalar field can propagate. In the relativistic case, this is the speed of light. In a non-relativistic system it is some characteristic speed of the system, for example the speed of second sound in superfluid helium-4. Correlations can never extend beyond a finite range, determined by a balance between the relaxation rate of the scalar field and the rate \dot{T}/T of the transition.

Zurek's argument may be paraphrased thus: As the system cools, ξ more or less keeps up, so long as it can do so, with the equilibrium correlation length, ξ_{eq} . But once $d\xi_{\text{eq}}/dt$ becomes larger than c , it can no longer do so. From then on, ξ does not change much until the point after the transition when it again becomes equal to the decreasing ξ_{eq} . That is the time, now often called the *Zurek time*, t_Z , at which we should identify ξ_{str} with ξ . An almost equivalent statement (up to a factor of order 1) is that t_Z is the time at which correlations starting from zero at the transition and propagating with speed c , can reach

the distance $\xi_Z = \xi_{\text{eq}}(t_Z)$.

This argument leads to a definite prediction for the defect density, l , that is, the average length of string per unit volume. At t_Z , we expect $l(t_Z) = k/\xi_Z^2$, where k is a constant roughly of order one. Numerical simulations [8,9] suggest that k is actually somewhat less than one, perhaps of order 0.1.

4 Tests in Liquid Crystals and Helium-4

Zurek [4] originally suggested testing these predictions by looking at a rapid quench in superfluid helium-4. Starting at high pressure in the normal phase just above the ‘lambda line’, a rapid reduction of pressure takes the sample through the transition into the superfluid phase.

The first experiments, however [10,11], were in thin samples of nematic liquid crystals, where networks of defects were seen to be formed when the system was rapidly cooled from the normal to the nematic phase. The nematic transition is first-order, so Zurek’s argument as given above is not directly applicable. In that case, what one expects [1] is that ξ_{str} is approximately (a few times) the mean distance between nucleation centres of bubbles of the new phase. This indeed seems to be the case.

The helium-4 experiment, which provides a more direct test of Zurek’s predictions, has now been performed, twice, by a group in Lancaster, using an apparatus comprising a small chamber filled with helium whose sides were formed of bellows so that it could be rapidly expanded. The object of the exercise was to detect the presence of any vortices formed during the transition by monitoring the absorption of second sound, which is strongly attenuated by vortices. The first experiment [12] did apparently show attenuation of the signal after the quench, falling off exponentially with a timescale of the order of a hundred milliseconds, as the vortices gradually disappeared, and compatible in magnitude with Zurek’s prediction for the number of vortices. But it was inconclusive for various reasons. Firstly, the detection equipment was swamped and unable to measure the attenuation until about 50 ms *after* the quench, so the inferred vortex density had to be extrapolated back to the Zurek time. Secondly, there were other possible sources of vorticity. In particular, vortices might have been nucleated by hydrodynamic effects at the bellows, and a particular concern attached to the capillary tube used to fill the chamber which was closed at its outer end, so that fluid was injected into the chamber on each expansion.

For these reasons, the Lancaster group decided to build an improved apparatus, in which the chamber was made much smoother, and in particular the

injection tube was closed at the point of entry into the chamber, to minimize the risk of extraneous vortex formation. However, results with this improved apparatus [13] rather disappointingly failed to reveal any vortex formation. I shall discuss possible reasons for this later.

5 Tests in Helium-3

Meanwhile, experiments were performed using helium-3 at two different laboratories, Helsinki and Grenoble (with collaboration from Lancaster). There are considerable advantages in using the lighter isotope. One is that the correlation length is much larger, say 40–100 nm as compared with less than 1 nm in ^4He . This means that a continuum description, of Ginzburg–Landau type, is a much better approximation. It also means that relatively speaking vortex formation requires much more energy, so that extraneous vortex formation is less likely. But perhaps the most important difference is that, because ^3He is an excellent neutron absorber, one can perform a temperature- rather than pressure-driven quench. Both experiments made use of this technique. The absorption of a neutron in a sample in the superfluid $^3\text{He-B}$ phase heats up a small region above the critical temperature. It then cools rapidly, in a period of about $1\ \mu\text{s}$, back through the transition into the superfluid phase. One expects the formation of a random tangle of vortices during this process.

In other respects the experiments were very different. The Helsinki experiment [14] used a sample at a temperature not far below the transition temperature, in a rotating cryostat. If the rotation speed v is less than some limit, no vortices are formed spontaneously at the walls, so the superfluid component is actually completely stationary, while the normal component is rotating with the container. The relative velocity between the two components means that any vortex formed following neutron absorption will be subject to the transverse *Magnus force*. If a vortex loop is big enough and appropriately oriented, the effect will be to expand it until it meets the walls of the container, after which the ends migrate to the top and bottom surfaces, and the vortex joins a central cluster parallel to the rotation axis.

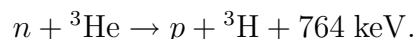
Detection of these captured vortices is made possible by another of the key features of ^3He , its non-zero nuclear spin, which makes it possible to use nuclear magnetic resonance. In the NMR trace one can actually see each individual vortex joining this central cluster. It turns out that the Zurek scenario leads to a very simple prediction for the dependence of the number of vortices captured per neutron event on v . There is a critical velocity v_c below which no vortices

are captured. When $v > v_c$,

$$n = \gamma \left\{ \left(\frac{v}{v_c} \right)^3 - 1 \right\},$$

where γ is a constant. Remarkably the entire dependence on the pressure, the bulk temperature and the magnetic field is contained in the single parameter v_c . The results clearly confirm this: plotted against v^3 , they fit well to straight lines, with a common intercept at $-\gamma$. Moreover the dependence of v_c on the bulk temperature also fits the prediction.

The Grenoble experiment [15] was complementary. Essentially it involved calorimetry. They used a sample at a much lower temperature, but in a non-rotating container, and sought to measure the energy release every time a neutron was absorbed. The absorption reaction is



The total energy released in the form of quasiparticles was measured, and found to peak in the range 575–650 keV, depending on the pressure. Some energy, around 50 keV, is also released in the form of ultraviolet radiation. However, there is a very clear overall deficit, which is attributed to vortices. It is very difficult to think of any other mechanism of energy loss. Moreover, the magnitude and the dependence on pressure are entirely consistent with Zurek's prediction.

6 Tests in Superconductors

From the point of view of analogy with cosmological phase transitions, tests in superconductors are particularly important, because they provide an example of symmetry breaking in a *local gauge theory*.

At least two such experiments have in fact been performed, by a group at Technion. In the first [16], they used a 1 cm² thin film of the high-temperature superconductor, YBCO. The film was heated to above T_c by optical irradiation, and then allowed to cool back through the transition. To determine the number of flux tubes formed during this process, the experimenters measured the total flux using a SQUID close to the sample, but separated from it by a mylar sheet. To be precise, this process does not measure the number of fluxons, but only the net total flux, the number of fluxons minus the number of antifluxons. A straightforward application of Zurek's technique would lead to an estimate for the net flux of about 100, whereas Carmi and Polturak's

results [16] were consistent with zero, with an error of about 10. Moreover, they argue that because of the different behaviour of a gauge theory, the actual prediction should be higher, around 10^4 , in which case there is a very clear contradiction. However, this prediction does not seem to be on very firm ground.

The same group have done another, similar experiment involving a loop of semiconductor with a large number of Josephson junctions in series. In that case they do find definite evidence [17] of the trapping of magnetic flux during the cooling process. Here the loop becomes superconducting before being linked by the junctions, so one might expect the phases to be random. The experiments actually show a *larger* flux than predicted on that basis, suggesting that the phase differences are not uniformly distributed, though it is not entirely clear why.

Experiments have been suggested [18] using an annular Josephson junction, in which cooling can lead to the trapping of radial flux lines. A group in Salerno has already done some experiments with an apparatus of this kind, but the experimental parameters were not optimized for this particular type of measurement: the predicted average number of trapped quanta is less than unity. Future experiments along these lines should yield interesting results.

7 Theoretical Interpretation

Zurek's predictions of the numbers of defects formed are based on an admittedly rather crude argument, and it is certainly not surprising to find that it needs modification.

At first sight, the difference between the behaviour of helium-3 and helium-4 may seem surprising, though in fact theoretically they are very different types of system. There is a critical region below T_c but above the so-called *Ginzburg temperature*, within which there is a large population of transient thermal loops. This region is much wider in helium-4 than in helium-3. Work by Karra and Rivers [19] has shown that this has a marked effect on the Zurek predictions. On the other hand, there may be a more immediate reason for the failure of the helium-4 experiment to detect any vortices: they may simply not last long enough to have been seen. Vorticity does decay with time, with a lifetime that is not well known. The authors [12] estimated the lifetime on the basis of measurements of decay of turbulent vorticity, but this may well be inapplicable [20,21]. It would be very desirable to measure the vorticity at a much earlier time after the transition. The situation should be clarified by future experiments [22].

Understanding the results in superconductors also presents a challenge. It is puzzling that the experiment with a thin film of YBCO saw nothing. One of the problems is that there is much more theoretical uncertainty about the predicted defect density in the case of a local gauge theory. There is a competing mechanism for the formation of defects, namely the thermal fluctuations in the magnetic field. Hindmarsh and Rajantie [23] have shown that that this mechanism yields a very different density and distribution of defects. It is unclear which mechanism will dominate. Neither mechanism seems to support the estimate that led to the conclusion of a disagreement by a factor of 1000 in the experiment with a thin superconducting film, but there remains *at least* a factor of 10 that is hard to explain.

The Josephson junction results seem to be positive, but if anything show too large a trapped flux. Why there should be this difference between the two experiments with superconductors is again something of a puzzle. More experiments along these lines too are very desirable.

8 Conclusions

The results so far are distinctly confusing. By far the best evidence in favour of the Zurek scenario comes from the experiments in helium-3, both of which yielded positive results that are hard to interpret in any other way. Results in nematic liquid crystals are also positive, but not a direct test of the Zurek scenario. In superconductors, it is puzzling that the experiment with a thin YBCO film saw no fluxons, while more than expected were seen in the Josephson-junction array.

It does seem that vortex formation in a rapid transition is now well established, but there is still a lot of work to do to understand the details.

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